# Exercise 46

Show that $\dfrac{d}{dx}\sqrt[4]{\dfrac{1+\tanh x}{1-\tanh x}} = \dfrac{1}{2}e^{x/2}$.

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### Solution

Simplify the function before taking the derivative for convenience.

$$\begin{aligned}
\frac{d}{dx}\sqrt[4]{\frac{1+\tanh x}{1-\tanh x}} &= \frac{d}{dx}\sqrt[4]{\frac{1+\frac{\sinh x}{\cosh x}}{1-\frac{\sinh x}{\cosh x}}} \\
&= \frac{d}{dx}\sqrt[4]{\frac{1+\frac{\sinh x}{\cosh x}}{1-\frac{\sinh x}{\cosh x}} \times \frac{\cosh x}{\cosh x}} \\
&= \frac{d}{dx}\sqrt[4]{\frac{\cosh x+\sinh x}{\cosh x-\sinh x}} \\
&= \frac{d}{dx}\sqrt[4]{\frac{\left(\frac{e^x+e^{-x}}{2}\right)+\left(\frac{e^x-e^{-x}}{2}\right)}{\left(\frac{e^x+e^{-x}}{2}\right)-\left(\frac{e^x-e^{-x}}{2}\right)}} \\
&= \frac{d}{dx}\sqrt[4]{\frac{\frac{1}{2}e^x+\frac{1}{2}e^{-x}+\frac{1}{2}e^x-\frac{1}{2}e^{-x}}{\frac{1}{2}e^x+\frac{1}{2}e^{-x}-\frac{1}{2}e^x+\frac{1}{2}e^{-x}}} \\
&= \frac{d}{dx}\sqrt[4]{\frac{e^x}{e^{-x}}} \\
&= \frac{d}{dx}\sqrt[4]{e^{2x}} \\
&= \frac{d}{dx}(e^{2x/4}) \\
&= \frac{d}{dx}(e^{x/2}) \\
&= e^{x/2}\cdot\frac{d}{dx}\left(\frac{x}{2}\right) \\
&= e^{x/2}\cdot\left(\frac{1}{2}\right) \\
&= \frac{1}{2}e^{x/2}
\end{aligned}$$